Study Material of B.Sc. III Semester

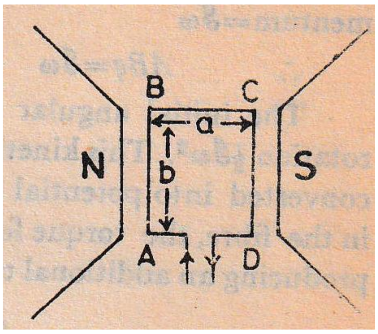
Electricity and Magnetism

Prepared by: Dr. Tanveer Hasan, Deptt. of Physics, Shia P.G. College, Lucknow, India.

Unit-III (LU)

The moving coil ballistic galvanometer

A moving coil ballistic galvanometer is a suspended type galvanometer which measures the charge rather than current.



Figure

A typical moving coil ballistic galvanometer is shown in adjacent figure in which a coil of length 'a' and breadth 'b' having current 'I' is placed between the pole pieces of a permanent magnet having radial magnetism. A twisting couple is formed by the two opposite forces 'IbB' acting on two vertically opposite sides AB and CD. The torque acting on the coil due to the current 'I' is given by:

$$\tau = I\ A\ B\ Sin\ \alpha = = I\ A\ B\ ....(1)$$

Where A is the area of coil and B is the strength of magnetic field of the magnet and angle $\alpha=0$ i.e. sin $\alpha=1$ is placed as plane of the coil is always parallel to the direction of magnetic field. The forces acting on sides BC and DA will be zero as they are parallel to the magnetic field (F=BIL sin θ, here θ=0 so F=0). If the current is passed in the coil for a short interval of time 'dt' then the impulse on each side BC and DA will be 'IbB dt'. So the total impulse given to the coil when a current is allowed to pass for time 't' is given by;

$$\int_0^t IbB\ dt = bB \int_0^t Idt = Bbq\ ....(2)$$

The moment of this impulse i.e. the angular impulse about the axis of suspension fiber is given as: $= Bbq \times a = BAq$ ....(3) *(where A =axb is area of coil)*

If there are N number of turns in the coil then the effective are of the coil A= Nab.

Now we know that moment of impulse = Moment of Inertia x angular velocity= Angular momentum, so if I is the moment of inertia of the coil and 'ω' is the angular velocity just after momentary current passed through the coil then;

$$\text{angular momentum} = I\ \omega\ ....(4)$$

from eqns. (3) and (4) we have; BAq = I ω ……….(5)

The initial angular momentum 'Iω' corresponds to a

Rotational kinetic energy = '$\frac{1}{2}I\omega^2$ ……………(6)

which rotates the suspension fiber and is converted into potential energy. If 'C' is the torque is required to produce unit twist in the fiber then to produce twist 'θ' is 'C θ'. Then the work done by the coil to produce an additional twist 'dθ' will be given by;

$$dW = C\,\theta\, d\theta \quad \text{…………..(7)}$$

Total work done in twisting the fiber through maximum twist 'θ_0' is given as;

$$W = \int_0^{\theta_0} C\,\theta\, d\theta = \frac{1}{2} C\,\theta_0^2 \quad \text{………(8)}$$

If we neglect friction force in the rotation of fiber then from eqns. (6) and (8) we can write;

$$\frac{1}{2} I\omega^2 = = \frac{1}{2} C\,\theta_0^2$$

or $$\omega^2 = \frac{C\,\theta_0^2}{I} \quad \text{……………(9)}$$

Using eqn. (5) for replacing 'ω'; $$\frac{(BAq)^2}{I^2} = \frac{C\,\theta_0^2}{I}$$

So we are left with; $$q^2 = \frac{C^2}{A^2B^2}\frac{I}{C}\,\theta_0^2 \quad \text{……………(10)}$$

Now the time period of the torsional motion of the coil about the suspension fiber is given by;

$$T = 2\pi\sqrt{\frac{I}{C}} \quad \text{………………(11)}$$

From eqn. (11) we find; $$\frac{I}{C} = \frac{T^2}{4\pi^2} \quad \text{………..(12)}$$

Putting it in eqn.(11); $$q^2 = \frac{C^2}{A^2B^2}\frac{T^2}{4\pi^2}\,\theta_0^2$$

Or $$q = \frac{C}{AB}\frac{T}{2\pi}\theta_0 = K\,\theta_0 \quad \text{………(13)}$$

Where $\mathbf{K} = \frac{\mathbf{C}}{\mathbf{AB}}\frac{\mathbf{T}}{\mathbf{2\pi}}$ is called the ballistic reduction factor or simply ***ballistic constant*** of the galvanometer. The ratio '$\frac{\mathbf{C}}{\mathbf{AB}}$' is called ***current reduction factor.***

The equation (13) shows that if know the ballistic constant 'K' and the maximum throw in the galvanometer 'θ_0' or we have the knowledge of current reduction factor we can find the total charge passing through the galvanometer in a short interval of time. *This time must be* *short as compared to time period of the coil so that the charge flows through the whole coil before it starts to move from its initial position.*

Relation between the current and charge sensitivity

Charge sensitivity of the galvanometer is defined as the deflection in mm on a scale of 1 meter away from the mirror of the galvanometer when I micro coulomb charge is passed through it. From equation (13) we can write;

Charge sensitivity = $\frac{\theta_0}{q} = \frac{2\pi}{T}\frac{AB}{C}$ …………(14)

But since we know that the current sensitivity = $\frac{\theta_0}{I} = \frac{AB}{C}$ ………(15)

From these eqns.(14) and (15) we have;

Charge sensitivity = $\frac{2\pi}{T}$ x Current sensitivity …………….(16)

Figure of merit of the galvanometer is defined as the charge required for unit deflection and it is the reciprocal of charge sensitivity = $\frac{q}{\theta_0}$.

Conditions for a moving coil galvanometer to be 'dead beat' or 'ballistic'

While deriving eqn.(13) we ignored the damping of the coil. In actual practice there are two types of damping present in the galvanometer and these are as follows:

(i) Mechanical damping: These are of again two types one is viscosity or friction of air, and other is the elastic hysteresis of the suspension fibre.

(ii) Electromagnetic damping: Opposing currents induced in any neighbouring body of metal and frame of coil itself it is made of a metal.

There exists a retarding couple due to the damping which is directly proportional to the angular velocity of the coil and is represented as '$p\frac{d\theta}{dt}$', where 'p' is called the damping coefficient.

If the galvanometer is connected in closed circuit the rotation is opposed by the induced current. The e.m.f. produced in the coil is equal to the rate of change of flux in the coil i.e. 'BAω', ω is the angular velocity of the coil. The corresponding induced current is also proportional to 'ω' and inversely proportional to the resistance 'R' of the circuit including the coil. This induced current opposes the rotation of coil in accordance to the Lenz's law. The retarding couple may therefore be taken as '$\frac{m}{R}\frac{d\theta}{dt}$', where 'm' is the constant containing the total flux 'AB' linked with coil. So the total retarding couple acting on the coil is represented by '$\frac{d\theta}{dt}\left[p + \frac{m}{R}\right]$.

When the coil rotates, a restoring couple 'Cθ' acts on the coil, where 'θ' is the angle by which the suspension fiber is twisted. I 'I' is moment inertia of the moving system including coil and '$\frac{d^2\theta}{dt^2}$' is the angular acceleration of the coil produced due to angular impulse, then the equation of motion of the coil according to Newton's II law, is represented by;

$$I\frac{d^2\theta}{dt^2} = -C\theta - \frac{d\theta}{dt}\left[p + \frac{m}{R}\right]$$

$$I\frac{d^2\theta}{dt^2} + \left[p + \frac{m}{R}\right]\frac{d\theta}{dt} + C\theta = 0$$

or $$\frac{d^2\theta}{dt^2} + 2b\,\frac{d\theta}{dt} + k^2\theta = 0 \text{ .... (17)}$$

where; $$\frac{1}{I}\left[p + \frac{m}{R}\right] = 2b;\ \frac{C}{I} = k^2 \text{ ....(18)}$$

Let the solution of this second order differential equation is;

$$\theta = A\,e^{\alpha t}, \text{ ....(19)}$$

Differentiating w.r.t.'t' twice we get;

$$\frac{d\theta}{dt} = A\alpha\,e^{\alpha t};\ \frac{d^2\theta}{dt^2} = A\alpha^2 e^{\alpha t} \text{ ....(20)}$$

Putting these values in eqn. (17) we get;

$$A\alpha^2 e^{\alpha t} + 2b\,A\alpha\,e^{\alpha t} + k^2 A\,e^{\alpha t} = 0$$

$$\alpha^2 + 2b + k^2 = 0 \text{ ....(21)}$$

On solving; $$\alpha = -b \pm \sqrt{b^2 - k^2} \text{ ....(22)}$$

Putting it in eqn. (19) we get the solution as;

$$\theta = A_1\,e^{(-b+\sqrt{b^2 - k^2})t} + A_2\,e^{(-b-\sqrt{b^2 - k^2})t} \text{ .... (23)}$$

Where A_1 and A_2 are constants. Now following three cases are possible:

(i) when b> k; then two values of α are real and negative and the solution (23) becomes; $\theta = A_1\,e^{\alpha_1 t} + A_2\,e^{\alpha_2 t}$ ....(24)

In this case the deflection θ goes on decreasing as the time passes and the motion is called "***dead beat*** or "non-oscillatory". The galvanometer is said be ***over damped*** in this condition. The coil is made of thick copper wire and has smaller number of turns so as to have comparatively smaller resistance 'R'. It is wound round a metallic frame. The moment of inertia of the moving coil system is not large and the suspension is comparatively thicker. This makes the period of oscillations of the coil very small.

(ii) When b=k; then two values of α are equal to –b and eqn. (23) becomes;

$$\theta = (A_1 + A_2)\,e^{-bt} = A_3 e^{-bt} \text{ .... (25)}$$

The coil in this condition comes out be in rest position in a very short time and the galvanometer is said to be "***critically damped***" or just non-oscillatory.

(iii) When b<k; then two values of α are imaginary. Now putting $\sqrt{(b^2 - k^2)} = j\sqrt{(k^2 - b^2)}$ in eqn. (23) we get;

$$\theta = e^{-bt}[A_1 e^{(j\sqrt{(k^2-b^2)})t} + A_2 e^{(-j\sqrt{(k^2-b^2)})t}]$$

$\theta = e^{-bt}[A_1\{\cos\sqrt{(k^2 - b^2)}t + j \sin\sqrt{(k^2 - b^2)}t\} + A_2\{\cos\sqrt{(k^2 - b^2)}t - j \sin\sqrt{(k^2 - b^2)}t\}]$

$\theta = e^{-bt}[(A_1 + A_2)\{\cos\sqrt{(k^2 - b^2)}\}t + j(A_1 - A_2)\{\sin\sqrt{(k^2 - b^2)}t\}]$ ……(26)

Putting $(A_1 + A_2) = B \sin\beta$ and $j(A_1 - A_2) = B\cos\beta$ ………………………. (27)

$\theta = Be^{-bt}\sin[\sqrt{(k^2 - b^2)}t + \beta]$ ………………….(28)

This shows that the motion of the coil is ***"oscillatory or ballistic"*** due to the sinusoidal term with a time period given as;

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\omega}$$

The period of oscillation T must have a large value in order that the whole charge may pass through the galvanometer before the coil has appreciably moved from its initial position. So 'I' must be large and 'C' must be small. Since the time period of the coil as given in eqn. (11) is $T = 2\pi\sqrt{\frac{I}{C}}$. Now since the motion of the coil to be ballistic we have 'b<k' so we can write from eqn. (18);

$$b < k$$

$$\frac{1}{2I}\left[p + \frac{m}{R}\right] < \sqrt{\left(\frac{C}{I}\right)}$$

$$\frac{1}{2I}\left[p + \frac{m}{R}\right] < \frac{2\pi}{T}$$

Therefore in order that the above condition satisfies 'b' must be small i.e. for this; (i)'I' must be large (ii) 'C' must be small which means the suspension fiber must be thin as possible (iii) 'p' i.e. air resistance must be small as possible (iv) 'm' should be small i.e. the coil must be wrapped on non-metallic frame like bamboo or paper or plastic (v) 'R' should be large i.e. the resistance of the coil and circuit should be large for this large number of turns of fine wire should be wrapped on the coil.

Correction of the amplitude of oscillation for damping: Logarithmic decrement

Now considering the equation (28) we have;

$$\theta = Be^{-bt}\sin[\sqrt{(k^2 - b^2)}t + \beta]$$

Since in the case of ballistic galvanometer 'b' is very small but still not zero, so the period of oscillation goes on decreasing with time as shown below;

The amplitude of oscillation at a time $t = Be^{-bt}$

At t=0; the amplitude of oscillation = $\theta_0 = B$

At $t = \frac{T}{4}$; ……………………….= $\theta_1 = Be^{-b\frac{T}{4}}$

At $t = \frac{3T}{4}$ ……………………….= $\theta_2 = Be^{-b\frac{3T}{4}}$

At $t = \frac{5T}{4}$ ……………………….= $\theta_3 = Be^{-b\frac{5T}{4}}$

At $t = \frac{7T}{4}$ ……………………….= $\theta_4 = Be^{-b\frac{7T}{4}}$

and so on.

Now we can see that; $\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} = \ldots\ldots\ldots\ldots\ldots\ldots = e^{-b\frac{T}{2}} = e^{-b\frac{\pi T}{\sqrt{(k^2-b^2)}}} = e^{\lambda} = d$ .....(29)

Equation number (29) shows that amplitude of successive oscillations continuously decreases and the ration of two consecutive amplitudes is constant 'd' which is termed as 'decrement' and 'λ' which is equal to '$\log_e$ d' is called 'logarithmic decrement. It is a constant for a galvanometer which is directly proportional to damping and depends upon the resistance of the galvanometer.

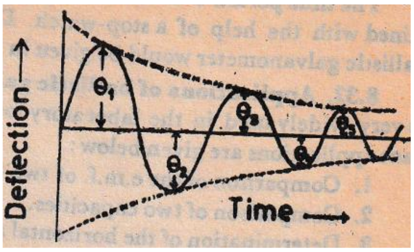


Figure 2

In the absence of damping the first (which is first maximum deflection) throw will be 'θ_1' which is after time $\frac{T}{4}$ second. Now since at t=0 in mean position we have θ_0= B; so now we have; $\theta_1 = Be^{-b\frac{T}{4}} = \theta_0 e^{-b\frac{T}{4}} = \theta_0 e^{-\frac{\lambda}{2}}$

So $\frac{\theta_0}{\theta_1} = e^{\frac{\lambda}{2}} = (1 + \frac{\lambda}{2} + \frac{\lambda^2}{4\,x2!} + \ldots\ldots\ldots)$

As 'λ' is very small so we can neglect second and higher order of it. So we are left with;

$$\frac{\theta_0}{\theta_1} = e^{\frac{\lambda}{2}} = (1 + \frac{\lambda}{2})$$

or; $\theta_0 = \theta_1 (1 + \frac{\lambda}{2})$ ....(30);

Putting this value in equation number (13) we have as follows;

$$q = \frac{C}{AB}\frac{T}{2\pi}\theta_0 = \frac{C}{AB}\frac{T}{2\pi}\theta_1 (1 + \frac{\lambda}{2}) \ldots.(31)$$

We can find 'λ' by recording first and eleventh throws. Where 'θ_1', θ_2, θ_3,.......... θ_{11}, are successive throws on either side of zero, then;

$$\frac{\theta_1}{\theta_{11}} = \frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} = \ldots\ldots\ldots\ldots\ldots\ldots \frac{\theta_{10}}{\theta_{11}} = e^{10\lambda}$$

or; $\lambda = \frac{1}{10}\log_e \frac{\theta_1}{\theta_{11}} = \frac{2.303}{10}\log_{10}(\frac{\theta_1}{\theta_{11}})$ ....(32)

By calculating 'λ' we can calculate the correction factor $(1 + \frac{\lambda}{2})$ and hence the corrected deflection θ_0.

(Reference Book: Electricity and Magnetism by Ahmad & Lal)