Study Material of B.Sc. III Semester

Electricity and Magnetism

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Unit-III (LU)

Mutual Inductance

When two circuits are placed near to each other, then if current changes in first circuit (called primary circuit) than an induced e.m.f. is produced in the second circuit (called secondary circuit) which gives rises to an induced current to flow in secondary when it is closed. This phenomenon is known as "Mutual Induction". Reverse phenomenon is also true. induced current also flows when there is a relative change in separation between two circuits is produced.

The magnitude of e.m.f. produced in the second circuit depends upon the amount of flux change linked with secondary due to change in current in primary. If 'I1' current flows in primary circuit due to which a part of flux produced due to it is linked with secondary is 'ϕ_{21}' then we can write:

$$\phi_{21} \propto I_1$$

$$\phi_{21} = M_{21} I_1 \text{ ....(1)}$$

Where M_{21} is called 'coefficient of mutual induction' between the circuits, which is defined as flux linked with secondary when unit flows in primary. The amount of flux linked from primary to secondary depends upon geometry of two circuits and the permeability of medium where two circuits are placed.

There will be flux linked with primary 'ϕ_{12}' due to an induced current I_2 in the secondary which is written as:

$$\phi_{12} = M_{12} I_2 \text{ ....(2)}$$

the coefficient of mutual inductions M_{12} and M_{21} are same we can write:

$$M_{12} = M_{21} = M \text{ ....(3); so in general we can write;}$$

$$\phi = M I \text{ ....(4)}$$

Where 'ϕ' is flux linked with second circuit when current 'I' flows in primary circuit. So when a current I changes in primary then an induced e.m.f. produced in the secondary is given by;

$$\xi = -\frac{\partial\phi}{\partial t} = -\frac{\partial(MI)}{\partial t} = -M\frac{\partial I}{\partial t} \text{ ....(5)}$$

So coefficient of mutual inductance may also defined as 'induced e.m.f. produced in secondary when a unit rate of change of current is made in primary circuit'.

The mutual-inductance of two circuits is related to their self-inductance L_1 and L_2. If 'I_1' be the current through primary then the flux 'ϕ_1' linked with it is given by:

$\phi_1 = L_1 I_1$……………….(6)

a part of this flux is linked with secondary circuit le t it be ϕ_{21} then:

$\phi_{21} = k\,\phi_1 = k L_1 I_1$ ……………(7)

Where 'k' is coefficient of coupling between two circuits. But we know that:

$\phi_{21} = M_{21} I_1$ ……………(8) ;

so we can write from eqns. (7) & (8) as;

$M_{21} I_1 = k L_1 I_1$

$M_{21} = k L_1$ …………..(9)

Similarly if 'I_2' is current in the secondary circuit and ϕ_2 is the flux linked with it then we have:

$\phi_2 = L_2 I_2$……………….(10)

A part of this flux is linked with the first circuit let it be ϕ_{12} then:

$\phi_{12} = k\,\phi_2 = k L_2 I_2$ ……..(11)

But we know: $\phi_{12} = M_{12} I_2$

$M_{12} I_2 = k L_2 I_2$

So; $M_{12} = k L_2$ …………..(12)

Now multiplying eqns. (11) & (12);

$M_{12} \times M_{21} = k L_2 \times k L_1$ ....(13)

Since the coefficient of mutual inductance between two coils is same so we can write

$M_{12} = M_{21} = M$; so eqn. (13) can be written as;

$M^2 = k^2 L_1 L_2$

$$\mathbf{M = k\sqrt{L_1 L_2}} \quad \text{..............(14)}$$

Here 'k' can take the values between 0 and 1 depending upon the geometry of two circuits and the permeability of the medium between them. For k=1 means total flux of one circuit is linked with the second circuit. For k=0 means there is no flux linkage between two circuits.

Combinations of Inductances

Inductance coils can be connected in two ways, namely (i) when there is no mutual inductance between them and (ii) When mutual inductance existing between them.

1. No mutual inductance

If 'n' inductance coils are so connected in series that there is no mutual inductance between them, then the induced e.m.f. across such a combination is:

$$\xi = -L_1 \frac{dI_1}{dt} - L_2 \frac{dI_2}{dt} - L_3 \frac{dI_3}{dt} \ldots\ldots\ldots\ldots - L_n \frac{dI_n}{dt}$$

$$\xi = -L \frac{dI}{dt} \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

The combined inductance of a coils connected in series is given by;

$$\mathbf{L = L_1 + L_2 + L_3 + \ldots\ldots\ldots L_n} \ldots\ldots\ldots\ldots\ldots(2)$$

If the coils are connected in parallel then the induced the e.m.f. is represented by;

$$\xi = -L_1 \frac{dI_1}{dt} = -L_2 \frac{dI_2}{dt} = -L_3 \frac{dI_3}{dt} = -L_n \frac{dI_n}{dt} \ldots\ldots\ldots\ldots\ldots(3)$$

Where I_1, I_2, I_3,... ... I_n are the current induced in the first, second etc. coils and I is the total induced current.

But $$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} + \frac{dI_3}{dt} \ldots \ldots + \frac{dI_n}{dt}$$

So $$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots\ldots\ldots + \frac{1}{L_n} \ldots\ldots\ldots\ldots(4)$$

This gives the effective inductance L of n coils connected in parallel.

2. Mutual inductance between the coils is present

If two inductance coils are connected in series and the mutual inductance between them is M then if the flux produced by the first coil and the flux that links it from the second coil are, at any instant, *in the same direction*, then the effective inductance of the first coil will be L_1+M. Similarly if the flux produced by the second coil and the flux that links it from the first coil are, at any instant, *in the same direction* then the effective inductance of the second coil will L_2+M. The total self inductance of the combination will be;

$$L = L_1 + M + L_2 + M = L_1 + L_2 + 2M \ldots\ldots\ldots\ldots(1)$$

When the flux produced by any coil and the flux that links it from the other coil is in *opposite directions* then,

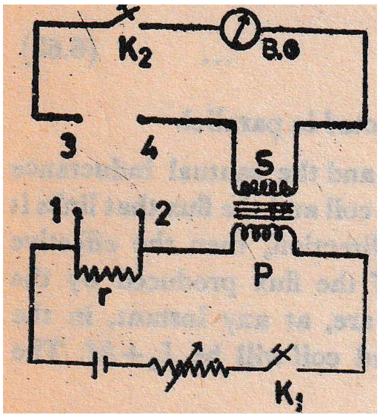
$$L = L_1 + L_2 - 2M \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

In general, the self inductance of the two coils will be,

$$\mathbf{L = L_1 + L_2 \pm 2M} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Measurement of Mutual Inductance

As shown in the adjacent circuit diagram 'P' and 'S' are the primary and secondary coils respectively of the given mutual inductance. Initially the terminal 1 is connected to 2 and terminal 3 to 4 by copper strips so that there is no connection between the primary and secondary circuits. The current in the primary circuit is adjusted to a suitable value using a rheostat. Now if key K_2 of the secondary circuit is closed first and then the key K_1 of the primary circuit, a throw is observed in the ballistic galvanometer. The key K_2, should however be released immediately after closing the key K_1 in order to avoid excessive damping of the galvanometer. The ballistic throw is set up by the induced e.m.f., produced due in the secondary circuit due to the sudden change in flux linked with the secondary circuit S, when the current in primary circuit P is established by pressing the key K_1. The induced produced in the secondary is represented by:



Figure

$$\xi = -M \frac{dI}{dt} \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Where $\frac{dI}{dt}$ is the rate of change of flux in the primary coil. So the induced current developed in the secondary is given by:

$$i = \frac{\xi}{R} = -\frac{M}{R}\frac{dI}{dt} \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Where 'R' is the total charge of the secondary circuit including the ballistic galvanometer. The total charge that flows through the secondary circuit in time 't' during which the current in the primary circuit attains the maximum value I_0, will be represented by:

$$q = \int_0^t i\,dt = \frac{M}{R}\int_0^t \frac{dI}{dt}\,dt = \frac{M}{R}\int_0^{I_0} dI = \frac{M}{R} I_0 \ldots\ldots\ldots(3)$$

Here we have ignored the negative sign because it tells only the direction of current. If the flow of the charge q produces a ballistic throw θ in the galvanometer, then we can write the charge in B.G as;

$$q = K \frac{T}{2\pi}\theta \left[1+\frac{\lambda}{2}\right] \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Equating equations (3) and (4);

$$\frac{M}{R} I_0 = K \frac{T}{2\pi}\theta \left[1+\frac{\lambda}{2}\right] \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

To find the value of K we find steady deflection in B.G, for this we connect terminal 1 with 3 and terminal 2 with 4. Now we introduce a small value of ‘r’ between 1 & 2 so that potential drop across r i.e ‘r I_0’may effect I_0 appreciably. This drop ‘r I_0’ serves as source of e.m.f. for the secondary circuit and when key K_1 is pressed first and then key K_2 a steady deflection θ_0 in B.G. is recorded due to the flow of current in secondary. We can write:

$$\frac{rI_0}{R} = K\,\theta_0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Dividing equations (5) and (6);

$$\frac{M}{r} = \frac{T}{2\pi}\,\frac{\theta}{\theta_0}\,[1+\frac{\lambda}{2}]$$

$$\text{or } M = r\,\frac{T}{2\pi}\,\frac{\theta}{\theta_0}\,[1+\frac{\lambda}{2}] \quad \ldots\ldots\ldots\ldots\ldots..(7)$$

The time period T can be calculated by pressing key K_2 first and then key K_1 in open circuit case i.e. terminals 1 with 2 and 3 with 4 are connected. We record time for ten oscillations and T can be obtained. Logarithmic decrement can be determined by pressing key K_2 first and then key K_1 and by recording the first throw and eleventh throw positions when terminal 1was connected with 3 and 2 with 4. Finally the value of coefficient of mutual inductance M can be calculated with eqn. (7).

(Reference Book: Electricity and Magnetism by Ahmad & Lal)