

# SHIA (POST-GRADUATE) COLLEGE, LUCKNOW

## DEPARTMENT OF MATHEMATICS

### **B.A./B.Sc. (Semester I)** **Mathematics** **(2020-21)**

#### **Paper I (Differential Calculus)**

##### **Unit I**

Definition of a sequence, Theorems on limits of sequences, Bounded and Monotonic sequences, Cauchy's convergence criterion, Cauchy sequence, limit superior and limit inferior of a sequence, subsequence, Series of non-negative terms, convergence and divergence, Comparison tests, Cauchy's integral test, Ratio tests, Root test, Raabe's logarithmic, de Morgan and Bertrand's tests, Alternating series, Leibnitz's theorem, Absolute and conditional convergence.

##### **Unit II**

Limit, Continuity and differentiability of function of single variable, Cauchy's definition, Heine's definition, equivalence of definition of Cauchy and Heine, Uniform continuity, Borel's theorem, boundedness theorem, Bolzano's theorem, Intermediate value theorem, Extreme value theorem, Darboux's intermediate value theorem for derivatives, Chain rule, Indeterminate forms.

##### **Unit III**

Successive differentiation, Leibnitz theorem, Maclaurin's and Taylor's series, Rolle's theorem, Lagrange and Cauchy Mean value theorems, Mean value theorems of higher order, Taylor's theorem with various forms of remainders, Partial differentiation, Euler's theorem on homogeneous function.

##### **Unit IV**

Tangent and Normals, Asymptotes, Curvature, Envelops and evolutes, Tests for concavity and convexity, Points of inflexion, Multiple points, Tracing of curves in Cartesian and Polar forms.

## **Paper II (Integral Calculus)**

### **Unit I**

Definite integrals as limit of the sum, Riemann integral, Integrability of continuous and monotonic functions, Fundamental theorem of integral calculus, Mean value theorems of integral calculus, Differentiation under the sign of Integration.

### **Unit II**

Improper integrals, their classification and convergence, Comparison test,  $\mu$ -test, Abel's test, Dirichlet's test, quotient test, Beta and Gamma functions, properties and convergence

### **Unit III**

Rectification, Volumes and Surfaces of Solid of revolution, Pappus theorem, Multiple integrals, change of order of double integration, Dirichlet's theorem, Liouville's theorem for multiple integrals

### **Unit IV**

Vector Differentiation, Gradient, Divergence and Curl, Normal on a surface, Directional Derivative, Vector Integration, Theorems of Gauss, Green, Stokes and related problems

# SHIA (POST-GRADUATE) COLLEGE, LUCKNOW

## DEPARTMENT OF MATHEMATICS

**B.A./B.Sc. (Semester II)**

**Mathematics**

**(2020-21)**

**Paper I (Matrices & Differential Equations)**

### **Unit I**

Types of Matrices, Elementary operations on Matrices, Rank of a Matrix, Echelon form of a Matrix, Normal form of a Matrix, Inverse of a Matrix by elementary operations, System of linear homogeneous and non-homogeneous equations, Theorems on consistency of a system of linear equations.

### **Unit II**

Eigen values, Eigen vectors and characteristic equation of a matrix, Caley-Hamilton theorem and its use in finding inverse of a matrix, Complex functions and separation into real and imaginary parts, Exponential and Logarithmic functions Inverse trigonometric and hyperbolic functions,

### **Unit III**

Formation of differential equations, Geometrical meaning of a differential equation, Equation of first order and first degree, Equation in which the variables are separable, Homogeneous equations, Exact differential equations and equations reducible to the exact form, Linear equations.

### **Unit IV**

First order higher degree equations solvable for  $x$ ,  $y$ ,  $p$ , Clairaut's equation and singular solutions, orthogonal trajectories, Linear differential equation of order greater than one with constant coefficients, Cauchy- Euler form.

## **Paper II (Geometry)**

### **Unit I**

General equation of second degree, System of conics, Tracing of conics, Confocal conics, Polar equation of conics and its properties.

### **Unit II**

Three-Dimensional Coordinates, Projection and Direction Cosine, Plane (Cartesian and vector form), Straight line in three dimension (Cartesian and vector form).

### **Unit III**

Sphere, Cone and Cylinder.

### **Unit IV**

Central conicoids, Paraboloids, Plane section of conicoids, Generating lines, Confocal conicoids, Reduction of second degree equations.

# SHIA (POST-GRADUATE) COLLEGE, LUCKNOW

## DEPARTMENT OF MATHEMATICS

### B.A./B.Sc. III Sem Mathematics (2020-21)

#### Paper I (Algebra)

##### Unit I

Equivalence relations and partitions, Congruence modulo  $n$ , Definition of a group with examples and simple properties, Subgroups, Generators of a group, Cyclic groups.

##### Unit II

Permutation groups, Even and odd permutations, The alternating group, Cayley's theorem, Direct products, Coset decomposition, Lagrange's theorem and its consequences, Fermat and Euler theorems

##### Unit III

Normal subgroups, Quotient groups, Homomorphism and isomorphism, Fundamental theorem of homomorphism, Theorems on isomorphism.

##### Unit IV

Rings, Subrings, Integral domains and fields, Characteristic of a ring, Ideal and quotient rings, Ring homomorphism, Field of quotient of an integral domain.

#### Paper II: Mathematical Methods

##### Unit I

Limit and Continuity of functions of two variables, Differentiation of function of two variables, Necessary and sufficient condition for differentiability of functions two variables, Schwarz's and Young theorem, Taylor's theorem for functions of two variables with examples, Maxima and minima for functions of two variables, Lagrange multiplier method, Jacobians.

## **Unit II**

Existence theorems for Laplace transforms, Linearity of Laplace transform and their properties, Laplace transform of the derivatives and integrals of a function, Convolution theorem, inverse Laplace transforms, Solution of the differential equations using Laplace transforms.

## **Unit III**

Fourier series, Fourier expansion of piecewise monotonic functions, Half and full range expansions, Fourier transforms (finite and infinite), Fourier integral.

## **Unit IV**

Calculus of variations-Variational problems with fixed boundaries- Euler's equation for functionals containing first order derivative and one independent variable, Extremals, Functionals dependent on higher order derivatives, Functionals dependent on more than one independent variable, Variational problems in parametric form.

# SHIA (POST-GRADUATE) COLLEGE, LUCKNOW

## DEPARTMENT OF MATHEMATICS

### **B.A./B.Sc. IV Sem** **Mathematics** **(2020-21)**

#### **Paper I: Differential Equations**

##### **Unit I**

Second order linear differential equations with variable coefficients: Use of a known solution to find another, normal form, method of undetermined coefficient, variation of parameters, Series solutions of differential equations, Power series method.

##### **Unit II**

Bessel, Legendre and hypergeometric functions and their properties, recurrence and generating relations.

##### **Unit-III**

Origin of first order partial differential equations. Partial differential equations of the first order and degree one, Lagrange's solution, Partial differential equation of first order and degree greater than one. Charpit's method of solution, Surfaces Orthogonal to the given system of surfaces.

##### **Unit-IV**

Origin of second order PDE, Solution of partial differential equations of the second and higher order with constant coefficients, Classification of linear partial differential equations of second order, Solution of second order partial differential equations with variable coefficients, Monge's method of solution.

## **Paper II: Mechanics**

### **Unit I**

Frame of reference, work energy principle, Forces in three dimensions, Poincot's central axis, Wrenches, Null lines and planes.

### **Unit II**

Virtual work, Stable and Unstable equilibrium, Catenary, Catenary of uniform strength

### **Unit III**

Velocities and accelerations along radial and transverse directions, and along tangential and normal directions, Simple Harmonic motion, Motion under other law of forces. Elastic strings, Motion in resisting medium, Constrained motion, Motion on smooth and rough plane curves.

### **Unit IV**

Motion of particles of varying mass, Rocket motion, Central orbit, Kepler's laws of motion,, Motion of particle in three dimensions, Rotating frame of reference, Rotating Earth, Acceleration in terms of different coordinates systems.



# SHIA (POST-GRADUATE) COLLEGE, LUCKNOW

## DEPARTMENT OF MATHEMATICS

### **B.A./B.Sc. (Semester V)**

### **Mathematics**

### **(2020-21)**

#### **Paper I: Numerical Analysis**

##### **Unit I**

Solution of equations: bisection, Secant, Regular Falsi, Newton Raphson's method, Newton's method for multiple roots, Interpolation, Lagrange and Hermite interpolation, Difference schemes, Divided differences, Interpolation formula using differences.

##### **Unit II**

Numerical differentiation, Numerical Quadrature: Newton Cotes Formulas, Gaussian Quadrature Formulas, System of Linear equations: Direct method for solving systems of linear equations (Gauss elimination, LU Decomposition, Cholesky Decomposition), Iterative methods (Jacobi, Gauss Seidel, Relaxation methods). The Algebraic Eigen value problem: Jacobi's method, Givens method, Power method.

##### **Unit III**

Numerical solution of Ordinary differential equations: Euler method, single step methods, Runge-Kutta method, Multi-step methods: Milne-Simpson method, Types of approximation: Last Square polynomial approximation, Uniform approximation, Chebyshev polynomial approximation.

##### **Unit IV**

Difference Equations and their solutions, Shooting method and Difference equation method for solving Linear second order differential equation with boundary conditions of first, second and third type.

#### **Paper II: Linear & Abstract Algebra**

##### **Unit I**

Automorphism, inner automorphism, automorphism groups and their computations, Conjugacy relations, Normaliser, Counting principle and the class equation of a finite group, Center of group of prime power order. Sylow's theorems.

##### **Unit II**

Prime and maximal ideals, Euclidean Rings, Principal ideal rings, Polynomial Rings, Polynomial over the Rational Field, The Eisenstein Criterion, Polynomial Rings over Commutative Rings, unique factorization domain.

##### **Unit III**

Vector spaces, Subspaces, Linear independence and dependence of vectors, Basis and Dimension, Quotient space, Linear transformations, The Algebra of linear transformations, rank nullity theorem, their representation as matrices.

#### **Unit IV**

Linear functionals, Dual space, Characteristic values, Cayley Hamilton Theorem, Inner product spaces, Cauchy-Schwarz inequality, Orthogonal vectors, Orthonormal sets and bases, Bessel's inequality for finite dimensional spaces, Gram-Schmidt orthogonalization process, Bilinear and Quadratic forms.

## **Paper III: Linear Programming**

#### **Unit I**

Linear programming problems, Slack and surplus variables, Standard and matrix forms of linear programming problem, Basic feasible solution.

#### **Unit II**

Convex sets, Fundamental theorem of linear programming, Simplex method.

#### **Unit III**

Artificial variables, Big-M method, Two phase method, Revised simplex method.

#### **Unit IV**

Duality in linear programming problems, Dual simplex method, Primal-dual method integer programming.

# SHIA (POST-GRADUATE) COLLEGE, LUCKNOW

## DEPARTMENT OF MATHEMATICS

### B.A./B.Sc. (Semester VI)

### Mathematics

(2020-21)

### Paper I : Analysis

#### Unit I

Definition and examples of metric spaces, Neighborhoods, Interior points, Limit Points, Open and closed sets, Convergent and Cauchy sequences, Completeness, Cantor's intersection theorem. Uniform convergence of sequences and series of functions, Uniform convergence and continuity, Uniform convergence and integration, Uniform convergence and differentiation, Power series.

#### Unit II

Stereographic projection, Continuity and Differentiability of complex functions, Analytic functions, Cauchy Riemann equations, Harmonic functions.

#### Unit III

Complex integration, Cauchy-Goursat theorem, Cauchy's Integral formula, Formulae for first, second and nth derivatives, Cauchy's Inequality, Maximum Modulus Theorem, Liouville's Theorem, Elementary functions, Mapping by elementary functions, conformal mapping.

#### Unit IV

Taylor and Laurent Series, Absolute and uniform convergence of Power series, Residues and Poles, Residue theorem, Zeros and poles of order  $m$ , Evaluation of improper real integrals, Definite integrals involving sines and cosines

### Paper II: Differential Geometry & Tensor Analysis

#### Unit I

Local theory of curves-Space curves, Examples, Plane Curves, tangent and normal and binormal, Osculating Plane, normal plane and rectifying plane, Helices, Serret-Frenet apparatus, contact between curve and surfaces, tangent surfaces, involutes and evolutes of curves, Bertrand curves, Intrinsic equations, fundamental existence theorem for space curves.

#### Unit II

Metric-first fundamental form and arc length, Direction coefficients, families of curves, intrinsic properties, geodesics, canonical geodesic equations, normal properties of geodesics, geodesics curvature, Gauss-Bonnet theorem, Gaussian curvature, normal curvature, Meusnier's theorem, mean curvature, Gaussian curvature, umbilic points, lines of curvature, Rodrigue's formula, Euler's theorem.

### **Unit III**

Tensor algebra: Vector spaces, the dual spaces, tensor product of vector spaces, transformation formulae, contraction, special tensor, inner product, associated tensor.

Tensor Analysis: Contravariant and covariant vectors and tensors, Mixed tensors, Symmetric and skew-symmetric tensors, Algebra of tensors, Contraction and inner product, Quotient theorem, Reciprocal tensors, Christoffel's symbols, Covariant differentiation.

### **Unit IV**

Gradient of scalars, Divergence of a contra-variant vector, covariant vector and conservative vectors, Laplacian of an invariant, curl of a covariant vector, irrotational vector, Riemannian space, Riemannian curvatures and their properties, Ricci tensor, and scalar curvature, Einstein space and Einstein tensor, Geodesics.

## **Paper III: Discrete Mathematics**

### **Unit I**

**Propositional Logic-** Proposition logic, basic logic, logical connectives, truth tables, tautologies, contradiction, normal forms (conjunctive and disjunctive), modus ponens and modus tollens, validity, predicate logic, universal and existential quantification, proof by implication, converse, inverse contrapositive, contradiction, direct proof by using truth table.

**Relation-** Definition, types of relation, domain and range of a relation, pictorial representation of relation, properties of relation, partial ordering relation.

### **Unit II**

**Boolean Algebra-** Basic definitions, Sum of products and products of sums, Logic gates and Karnaugh maps.

**Graphs-** Simple graph, multi graph, graph terminology, representation of graphs, Bipartite, regular, planar and connected graphs, connected components in a graph, Euler graphs, Hamiltonian path and circuits, Graph colouring, chromatics number, isomorphism and homomorphism of graphs.

### **Unit III**

**Combinatorics-** Inclusion- exclusion, recurrence relations (nth order recurrence relation with constant coefficients, Homogeneous recurrence relations, Inhomogeneous recurrence relations), generating function (closed form expression, properties of G.F., solution of recurrence relations using G.F. solution of combinatorial problem using G.F.)

### **Unit IV**

**Finite Automata-** Basic concepts of automation theory, Deterministic Finite Automation (DFA), transition function, transition table, Non Deterministic Finite Automata (NFA), Mealy and Moore machine, Minimization of finite automation.